Quiz 9.1: Sample Answers

1. Find the absolute maximum and minimum values of

$$f(x) = -4x - 4x^3 + 8x^2 - 1$$

on the interval $[0, 2]$.

We first find the derivative, and set it equal to 0 to find the critical points:

$$f'(x) = -4 - 12x^2 + 16x = 0$$

Dividing by -4, we get

$$3x^2 - 4x + 1 = 0$$

We then use the quadratic formula to find the solutions to this equation: $x = 1$, $x = \frac{1}{3}$. Finally, we find $f(x)$ at these critical values, as well as at the end points (0 and 2):

$$\begin{aligned} f(0) &= -1 \\ f\left(\frac{1}{3}\right) &= \frac{-43}{27} \\ f(1) &= -1 \\ f(2) &= -9 \end{aligned}$$

So the maximum is -1, and the minimum is -9.

2. Find the absolute maximum and minimum values of

$$f(x) = -2\sin x - 3\cos x$$

on the interval $[-\pi, \pi]$.

We first find the derivative, and set it equal to 0 to find the critical points:

$$f'(x) = -2\cos x + 3\sin x = 0$$

re-arranging gives

$$\frac{\sin x}{\cos x} = \frac{2}{3} \text{ or } \tan x = \frac{2}{3}$$

Thus the critical points are when $x = \arctan \frac{2}{3}$.

We need to evaluate $f(x)$ at $x = \arctan \frac{2}{3}$. Since this gives $\tan x = \frac{2}{3}$, the hypoteneuse of the triangle with angle x is $\sqrt{2^2 + 3^2} = \sqrt{13}$. Thus $\sin x = \frac{2}{\sqrt{13}}$ and $\cos x = \frac{3}{\sqrt{13}}$. Thus

$$f\left(\arctan \frac{2}{3}\right) = -2\frac{2}{\sqrt{13}} - 3\frac{3}{\sqrt{13}} = \frac{-13}{\sqrt{13}} = -\sqrt{13}$$

In addition, $f(-\pi) = 3$, and $f(\pi) = 3$.

Thus the maximum is 3, and the minimum $-\sqrt{13}$.

3. Find the absolute maximum and minimum values of

$$f(x) = e^{-3x} - e^{-4x}$$

on the interval $[0, 1]$.

We first find the derivative, and set it equal to 0 to find the critical points:

$$f'(x) = -3e^{-3x} + 4e^{-4x} = 0$$

Re-arranging this gives

$$4e^{-4x} = 3e^{-3x} \text{ or } e^{-x} = \frac{3}{4}$$

Thus we get $x = -\ln \frac{3}{4}$ is the only critical point.

Plugging in this value as well as the endpoints 0 and 1, we have

$$\begin{aligned} f(0) &= 0 \\ f(1) &= e^{-3} - e^{-4} \\ f(-\ln 3/4) &= (3/4)^3 - (3/4)^4 \end{aligned}$$

This last value is larger than $e^{-3} - e^{-4}$, so it is the max, while 0 is the min.

4. Find the absolute maximum and minimum values of

$$f(x) = x\sqrt{9 - x^2}$$

on the interval $[-1, 3]$.

We first find the derivative:

$$f'(x) = \frac{-x^2}{\sqrt{9 - x^2}} + \sqrt{9 - x^2}$$

First, the derivative does not exist for $|x| \geq 3$, since at ± 3 one is dividing by 0, and for $|x| > 3$, the square root is undefined. However, all of these values except 3 are outside of the interval, and 3 will already be considered as an endpoint value. The other critical points are when the derivative equals 0:

$$\begin{aligned} \frac{-x^2}{\sqrt{9 - x^2}} + \sqrt{9 - x^2} &= 0 \\ -x^2 + (9 - x^2) &= 0 \\ x^2 &= \frac{9}{2} \\ x &= \pm \frac{3}{\sqrt{2}} \end{aligned}$$

However, $\frac{-3}{\sqrt{2}}$ is outside outside of the bounds, so the only extra value we need to consider is $\frac{3}{\sqrt{2}}$.

We plug in all these values:

$$\begin{aligned} f(-1) &= -\sqrt{8} \\ f(3) &= 0 \\ f\left(\frac{3}{\sqrt{2}}\right) &= \frac{9}{2} \end{aligned}$$

Thus the maximum values is $\frac{9}{2}$, and the minimum $-\sqrt{8}$.